AIAA JOURNAL
Vol. 31, No. 8, August 1993

Phase Nonequilibrium Effects on the Gain of a Two-Phase Flow Gasdynamic Laser

R. K. Thulasiram* and N. M. Reddy†
Indian Institute of Science, Bangalore 560 012, India

Analysis of gas-particle nozzle flow is carried out with attention to the effect of dust particles on the vibrational relaxation phenomena and consequent effects on the gain of a gasdynamic laser. The phase nonequilibrium between the gas mixture and the particles during the nozzle expansion process is taken into account simultaneously. The governing equations of the two-phase nozzle flow have been transformed into similar form, and general correlating parameters have been obtained. It is shown from the present analysis that the particles present in the mixture affect the optimum gain obtainable from a gasdynamic laser adversely, and the effect depends on the size and loading of the particles in the mixture.

Nomenclature

A = area ratio of the nozzle
c = specific heat of the particle material
c_p = specific heat of the gas phase at constant pressure
d_p = diameter of the (spherical) particles
$(e_v)_{\mathrm{I}}, (e_v)_{\mathrm{II}}$ = vibrational energies of modes I and II of the CO_2–N_2 system
G_0 = small signal gain
i, j = nozzle shape parameters
L' = nozzle scale parameter, $r'_*/\tan\theta$
$\dot m, \dot m_p$ = mass flow rate of gas and particle phases
N_s = function introduced in Eqs. (25), (26), and (29)
Nu = Nusselt number
Re = Reynolds number (based on particle diameter)
r = nozzle throat radius
T = translational temperature of gas phase
$T_{\mathrm{I}}, T_{\mathrm{II}}$ = vibrational temperatures of modes I and II of the CO_2–N_2 system
T_p = temperature of the particle phase
u, u_p = velocities of gas and particle phases
X = mole fraction
x = distance along the nozzle axis
α = independent variable introduced for transformation
γ = specific heat ratio of the gas phase
δ = relative specific heat of the particles and gas, c/c_p
η = loading ratio, $\dot m_p/\dot m$
θ = angle of inclination of the nozzle wall from the axis
$\theta_1, \theta_2, \theta_3$ = characteristic vibrational temperatures of different modes of CO_2
θ_N = characteristic vibrational temperature of N_2
λ_ν = wavelength of the laser transition
$\lambda, \lambda_{\mathrm{I}}, \lambda_{\mathrm{II}}$ = correlating parameters
μ = viscosity of the gas phase
ξ = independent variable introduced for transformation
ρ, ρ_p = density of gas and particle phases
$\tau_{\mathrm{I}}, \tau_{\mathrm{II}}$ = characteristic relaxation times of modes I and II of the CO_2–N_2 system
τ_v = velocity relaxation time of particles, $\rho_p d_p^2/(18\mu)$
$\phi_{\mathrm{I}}, \phi_{\mathrm{II}}$ = normalized vibrational temperatures of the two modes of the CO_2–N_2 system
ψ = normalized translational temperature of the gas phase
ψ_p = normalized temperature of the particle phase
$\chi, \chi_{\mathrm{I}}, \chi_{\mathrm{II}}$ = general correlating parameters

Subscripts

C, N, H = CO_2, N_2, and He, respectively
p = pertaining to particles (except c_p)
0, * = reservoir and nozzle throat conditions, respectively

Superscripts

eq = equilibrium condition
$'$ = dimensional quantities

Introduction

THE idea of stimulated emission, although put forward by Albert Einstein[1] as early as 1917, was utilized only in 1961 by Javan et al.[2] to generate lasing action in gases. The significant effect of nonequilibrium among vibrational energy levels on the flow was demonstrated by Kantrowitz,[3] and lasing action in CO_2 was reported first by Patel.[4] The power output that could be obtained in a gas laser was limited by several factors, the first and foremost being arcing in the large-scale devices; and heating of the laser gas by the electric discharge, which destroys the population inversion (PI) among the energy levels and hence terminates the lasing action. These problems were obviated by the invention of gasdynamic lasers (GDLs).

A new method of creating PI by rapid heating or cooling of the system was suggested by Basov and Oraevskii.[5] Hurle and Hertzberg,[6] using gasdynamic means, suggested that one of the options—rapid cooling—could be obtained in the nonequilibrium expansion of an initially hot gas through a supersonic nozzle. A GDL was constructed and successfully demonstrated for the first time at AVCO Everett Research Laboratory by Gerry[7] with a CO_2–N_2–H_2O gas mixture. First, theoretical analysis was carried out for the calculation of PI in rapid expansion through a supersonic nozzle by Basov et al.[8] Anderson[9] introduced a two-mode vibrational kinetic model for a CO_2–N_2 system and carried out theoretical calculations with a mixture of CO_2–N_2–H_2O gases. The results[10] obtained from theory and experiments were found to agree well.

Presented as Paper 92-3019 at the AIAA 23rd Plasmadynamics and Lasers Conference, Nashville, TN, July 6–8, 1992; received Aug. 4, 1992; revision received Jan. 12, 1993; accepted for publication Feb. 19, 1993. Copyright © 1993 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Research Scholar, Department of Aerospace Engineering; currently Postdoctoral Fellow, Department of Mechanical Engineering, Concordia University, Montreal H3G 1M8, Canada. Member AIAA.

†Professor, Department of Aerospace Engineering. Associate Fellow AIAA.

This field of GDL saw jumps and leaps at a faster rate since the advent of the idea of obtaining PI by gasdynamical means. The technological developments in other fields suggested ideas and means of getting higher power output from a GDL. For example, Lee[11] obtained a 80-kW power output from a GDL using a combustion means of getting a highly excited laser gas mixture. Quite extensive reviews have been reported in books by Anderson[12] and Losev[13] and in papers by Anderson,[14] Christiansen et al.,[15] Murthy,[16] and Hertzberg.[17]

In a CO_2–N_2 GDL, the necessary pumping of the lasing gas (CO_2) is done by N_2. Thus, large vibrational energy is required in N_2 for contribution toward CO_2; that is, N_2 acts here as a donor gas. Hence, it is desirable to have as much energy in the donor gas as possible so that pumping of the higher energy levels of CO_2 is done very efficiently. But if the mixture is heated to high temperatures because of the relatively smaller dissociational characteristic temperature for CO_2, the lasing gas CO_2 will start dissociating. Thus, the gain (which is a direct measure of PI) and the specific energy of the GDL will be limited. Instead, if highly excited N_2 is obtained in some manner other than by means of heating the mixture, is mixed with cold CO_2, and then is expanded through the nozzle for the lasing action, not only high enough PI could be expected but also the dissociation problem of CO_2 could be avoided. It is a common practice to add He or H_2O in the mixture so that the lower energy levels of the lasing gas (CO_2) are depleted fast, thereby creating a significant amount of PI. In other words, pumping of higher energy levels of CO_2 is done by N_2, and the depletion of lower energy levels of CO_2 is done by He or H_2O.

Two-Phase Flow Gasdynamic Laser

As mentioned earlier, pumping of higher energy levels of CO_2 is done by highly excited N_2. One of the means of generating highly excited N_2 is by burning metal powder like aluminum or magnesium in air. The metal powder gets burnt during combustion and becomes metal oxide particles. In this case, the combustion products do not contain the contaminant gases like those produced by burning hydrocarbon fuels such as CH_4 and C_6H_6, and therefore the deterioration of the lasing efficiency will depend only on the metal oxide particles. These combustion products are mixed with cold CO_2 and then expanded through a nozzle for lasing action. The metal oxide particles are accelerated almost exclusively by the drag forces associated with lag (or slippage) of the particles relative to the expanding gas and hence will be deleterious to the effectiveness of the nozzle expansion process. Therefore, the velocity and temperature of the gas and particles have to be considered separately for the analysis. Moreover, since the particles are dragged out of the nozzle by the forces exerted by the gas, the gas will lose part of its energy. This way, the effective specific energy of the lasing gas (mixture) may be reduced. Therefore, for any accurate analysis this nozzle flow problem has to be treated as a two-phase flow consisting of gas (mixture) and particles. The effect of phase nonequilibrium in this kind of two-phase nozzle flow has to be taken into account appropriately for the analysis of internal energy relaxation phenomena leading to the study of lasing performance of a gas mixture. Such an effect of the particles on the performance of a GDL has been reported by Veselov and Markachev.[18] They have studied the problem with a different geometrical setup of nozzles in which the highly excited N_2 is mixed with cold CO_2 downstream of the nozzle exit, and hence they effectively solved the equations in a constant area duct.

Scope of the Present Work

Analysis of the gas-particle flow in a nozzle is complex because of the large number of parameters involved. Hence, optimization of the performance of the two-phase flow GDL with respect to every system parameter would be a formidable task requiring tedious and time-consuming procedures. However, a method has already been proposed[19] for gas-particle nozzle flows, and a general correlating parameter has been obtained. This correlating parameter combines all of the parameters of the problem, and a single value of this leads to several combinations of the constituting parameters. In the present investigation, this method has been extended to the problem of simultaneously (particle and vibrational) relaxing the CO_2–N_2–He gas-particle mixture to obtain similar solutions and correlating parameters for the two-phase flow GDL, and hence the optimum gain is calculated. Also, new expressions relating nozzle area ratio and local Mach number applicable to gas-particle mixtures derived earlier[19] by introducing the concept of virtual speed of sound have been used for the present analysis and are discussed briefly in the Appendix. In the present investigation, burning of metal powder in air is assumed not to produce any contaminant gases like O, CO, NO, etc., except metal oxide particles. Hence, no species are considered for the present study. Also, these particles are assumed to be inert except for the momentum and heat transfer processes. Deactivation of molecular energy due to collision of gas molecules with particles is not considered in the present analysis since this effect is expected to be a very slow and inefficient process due to large mass difference between the particles and the molecules of the gas phase. Furthermore, the necessary deactivation rate constants are not available in the literature.

Kinetic Model of a Gasdynamic Laser

In the present study the GDL mixture consists of CO_2, N_2, and He gases. The number of vibrational energy exchange reactions in such a system is very large, and best-known rates reported[20] earlier have been used for the required energy exchange processes. Based on certain observations of the molecular energy exchange reactions involved, Anderson et al.[10] and Anderson[12] have introduced an approximate two-temperature model for the vibrational kinetics of this CO_2–N_2 system that has been used in the present investigation. Figure 1 depicts a schematic of this vibrational energy storage model, including various vibrational levels and their energies. In Fig. 1, mode I (of vibrational temperature T_I) and mode II (of vibrational temperature T_{II}) include, respectively, the lower and upper energy levels of the system, and τ_a, τ_b, and τ_c are the average relaxation times of these modes.

The relaxation of the vibrational modes of the lasing gas (CO_2) is assumed to follow the simple harmonic oscillator model, and hence the vibrational relaxation equations are given by[12]

$$\frac{d(e_v')_I}{dt} = \frac{1}{\tau_I'}[(e_v')_I^{eq} - (e_v')_I] \tag{1}$$

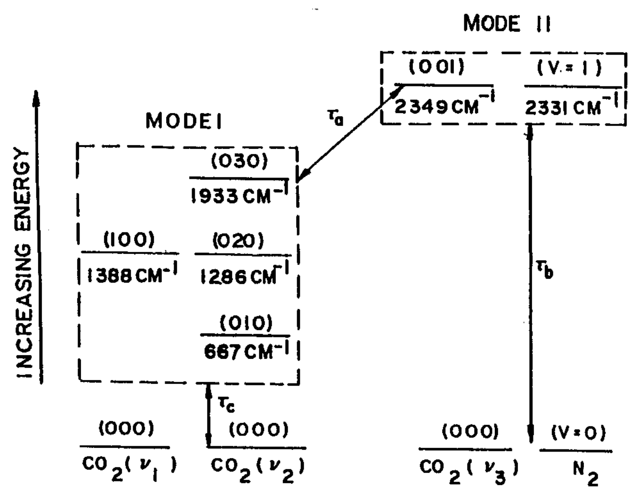

Fig. 1 Schematic of grouping of energy levels for the vibrational model of the CO_2–N_2 gasdynamic laser system.

$$\frac{d(e_v')_{II}}{dt} = \frac{1}{\tau_{II}'} [(e_v')_{II}^{eq} - (e_v')_{II}] \tag{2}$$

where t is the time. These are the averages that characterize the net rate of energy transfer into and out of modes I and II; this energy transfer is generally assumed to be governed by the major transitions, as shown by arrows in Fig. 1, which are identified with the relaxation times τ_a, τ_b, and τ_c. These relaxation times are themselves averages of the detailed CO_2-CO_2, CO_2-N_2, CO_2-He, N_2-N_2, and N_2-He collisions. Such averages for a mixture of gases are obtained from the "parallel resistance" mixture rule given[12] by

$$\frac{1}{\tau_a'} = \frac{X_C}{(\tau_a')_{C-C}} + \frac{X_N}{(\tau_a')_{C-N}} + \frac{X_H}{(\tau_a')_{C-H}} \tag{3}$$

$$\frac{1}{\tau_b'} = \frac{X_C}{(\tau_b')_{N-C}} + \frac{X_N}{(\tau_b')_{N-N}} + \frac{X_H}{(\tau_b')_{N-H}} \tag{4}$$

$$\frac{1}{\tau_c'} = \frac{X_C}{(\tau_c')_{C-C}} + \frac{X_N}{(\tau_c')_{C-N}} + \frac{X_H}{(\tau_c')_{C-H}} \tag{5}$$

The average relaxation times for modes I and II are obtained from

$$\tau_I' = \tau_c' \tag{6}$$

$$\frac{1}{\tau_{II}'} = \left(\frac{X_C}{\tau_a'} + \frac{X_N}{\tau_b'}\right)\left(\frac{1}{X_C + X_N}\right) \tag{7}$$

These values will be used in Eqs. (1) and (2). It is to be emphasized here that the simplified model represented by modes I and II in Fig. 1 and by Eqs. (1-7) are intended only for the calculation of the PI (and hence gain) in CO_2-N_2-He mixtures; it is not necessarily valid for other gases, nor can it be used when a substantial amount of radiative energy interaction within the gas is present.

To calculate the PI, the populations in different energy levels are needed. The populations N of the 001 and 100 levels in CO_2 are obtained from

$$N_{001}' = N_C' e^{-\theta_3'/T_{II}'}/Q_{vib} \tag{8}$$

$$N_{100}' = N_C' e^{-\theta_1'/T_I'}/Q_{vib} \tag{9}$$

where Q_{vib} is the partition function given[12] by

$$Q_{vib} = (1 - e^{-\theta_1'/T_I'})^{-1}(1 - e^{-\theta_2'/T_I'})^{-2}(1 - e^{-\theta_3'/T_{II}'})^{-1}$$

With the appropriate rotational constants, the gain for the $P(20)$ transition (i.e., the transition from the 19th to the 20th rotational level) of the 10.6-μm band is[12]

$$G_0 = \frac{\lambda_\nu'^2}{4\pi\tau_{21}'\nu_C'} (N_{001}' - N_{100}') \frac{45.6}{T'} e^{-234/T'} \tag{10}$$

where τ_{21}' is the spontaneous radiative lifetime equal to 5.38 s, ν_C' is the collision frequency, and λ_ν' is the wavelength of the $P(20)$ laser transition.

Analysis

The governing equations for the simultaneous particle and vibrational nonequilibrium gas mixture consist of equations that describe the vibrational mode relaxation in addition to the usual two-phase flow equations. The equations in nondimensional form for negligible volume fraction of particles are given by the following:

Mass:

$$\rho u A = \rho_* u_* \tag{11}$$

Momentum:

$$\rho u \frac{du}{dx} + \eta\rho u \frac{du_p}{dx} + \rho \frac{dT}{dx} + T\frac{d\rho}{dx} = 0 \tag{12}$$

Energy:

$$u\frac{du}{dx} + (1+\epsilon)\frac{dT}{dx} + \frac{de_v}{dx} + \eta\left(u_p\frac{du_p}{dx} + \frac{\gamma}{\gamma-1}\delta\frac{dT_p}{dx}\right) = 0 \tag{13}$$

where $\epsilon = 2.5(X_C + X_N) + 0.5X_H$,

Drag:

$$u_p\frac{du_p}{dx} = (u - u_p)\frac{f(Re)}{\tau_v} \tag{14}$$

Heat transfer:

$$u_p\frac{dT_p}{dx} = \frac{T - T_p}{2}\frac{1}{\delta}\frac{Nu(Re)}{\tau_v} \tag{15}$$

Vibrational rate equations:

$$u\frac{d(e_v)_k}{dx} = \frac{L'}{u_0'\tau_k'}[(e_v)^{eq} - e_v]_k, \qquad k = \mathrm{I, II} \tag{16}$$

The equation of state has been used in eliminating the pressure term from the mixture momentum equation. The vibrational energy has two parts, one due to mode II and the other due to mode I. The normalized specific energies of these modes are given[12] by

$$(e_v)_I = X_C E_I(T_I, \theta_1, \theta_2) \tag{17}$$

and

$$(e_v)_{II} = X_C E_{II}(T_{II}, \theta_3, \theta_N, X_N/X_C) \tag{18}$$

where

$$E_I = \frac{\theta_1}{e^{\theta_1/T_I} - 1} + \frac{2\theta_2}{e^{\theta_2/T_I} - 1}$$

$$E_{II} = \frac{\theta_3}{e^{\theta_3/T_{II}} - 1} + \frac{X_N}{X_C}\frac{\theta_N}{e^{\theta_N/T_{II}} - 1}$$

Therefore,

$$\frac{d(e_v)_I}{dx} = X_C G_I \frac{dT_I}{dx} \tag{19}$$

where

$$G_I = \frac{dE_I}{dT_I} = \left[\frac{e^{\theta_1/T_I}(\theta_1/T_I)^2}{(e^{\theta_1/T_I} - 1)^2} + 2\frac{e^{\theta_2/T_I}(\theta_2/T_I)^2}{(e^{\theta_2/T_I} - 1)^2}\right] \tag{20}$$

Thus, since $e_v = (e_v)_I + (e_v)_{II}$,

$$\frac{de_v}{dx} = \sum_{k=I}^{II}\frac{d(e_v)_k}{dx} = X_C\sum_{k=I}^{II} G_k\frac{dT_I}{dx} \tag{21}$$

In the present analysis, the nondimensionalized temperatures are normalized with reference to the characteristic vibrational temperature θ_N of nitrogen such that

$$T = \theta_N\psi; \qquad T_k = \theta_N\phi_k; \qquad k = \mathrm{I, II}$$
$$\theta_k = \theta_N\overline{\theta_k}, \qquad k = 1, 2, 3, \text{ and } N \tag{22}$$

That is, ψ and ϕ_k are the normalized translational and vibrational temperatures.

As has been mentioned earlier, the present study is an application of the method described in Ref. 19; a new variable α, $\alpha = -\ln(\rho)$, is introduced, and the governing equations using normalized temperatures with α as the independent variable can be written as

$$u\frac{du}{d\alpha} + \eta u\frac{du_p}{d\alpha} + \theta_N\frac{d\psi}{d\alpha} - \theta_N\psi = 0 \tag{23}$$

$$u\frac{du}{d\alpha}+(1+\epsilon)\theta_N\frac{d\psi}{d\alpha}+X_C\sum_{k=\mathrm{I}}^{\mathrm{II}}G_k\theta_N\frac{d\phi_k}{d\alpha}$$
$$+\eta\left(u_p\frac{du_p}{d\alpha}+\frac{\gamma}{\gamma-1}\delta\theta_N\frac{d\psi_p}{d\alpha}\right)=0 \tag{24}$$

$$\frac{du_p}{d\alpha}=\frac{e^{\alpha/ij}(\rho_*u_*)^{1/ij}}{N_s(ij)(uZ)^{1/ij}}\frac{u-u_p}{u_p}\frac{f(Re)}{\tau_v}$$
$$=\frac{e^{\alpha/ij+\lambda}}{N_s}\frac{u-u_p}{u_p}f(Re)(uZ)^{-1/ij} \tag{25}$$

$$\frac{d\psi_p}{d\alpha}=\frac{e^{\alpha/ij}(\rho_*u_*)^{1/ij}}{N_s(ij)(uZ)^{1/ij}}\theta_N\frac{\psi-\psi_p}{u_p}\frac{1}{\delta}\frac{Nu(Re)}{2\tau_v}$$
$$=\frac{e^{\alpha/ij+\lambda}}{N_s}\frac{1}{\delta}\frac{\psi-\psi_p}{u_p}\frac{Nu(Re)}{2}(uZ)^{-1/ij} \tag{26}$$

where

$$N_s=\frac{M_v^2}{M_v^2-1}(1-A_v^{-1/i})^{(j-1)/j}$$
$$\lambda=\ell n\left[\frac{(\rho_*u_*)^{1/ij}}{(ij)\tau_v}\right]$$

where A_v is the virtual area ratio given by $A_v=A/Z$, Z is a parameter representing nonequilibrium (between gas and particles) characteristics of the flow given by $Z=1+\eta u_p/u$, M_v is the virtual Mach number given by $M_v=u/a_v$ corresponding to virtual speed of sound a_v given by $a_v=\{(1/Z)(dp/d\sigma)\}^{1/2}$, p is the pressure, σ is the concentration of gas phase in the mixture given by $\sigma=(1-\zeta)\rho$, ζ is the volume fraction of the particles in the mixture, and i and j are the nozzle shape parameters with $i=1$ and $j=2$ representing a hyperbolic nozzle and $i=2$ and $j=1$ representing a conical nozzle. An expression for $(d\alpha/dx)$ given[19] by

$$\frac{d\alpha}{dx}=\frac{M_v^2}{M_v^2-1}(ij)(A_v)^{-1/ij}[1-(A_v)^{-1/i}]^{(j-1)/j}$$

is used to obtain Eqs. (25) and (26) in the present form. A brief discussion of the terms like A_v and M_v is presented in the Appendix. The term τ' in the vibrational rate Eqs. (16) can be presented[21] in the form $\tau'=W\exp[Y/(T')^{1/3}](p')^{-1}$ where W and Y are constants given by $W_{CC}=1.555\times10^{-8}$ atm-s, $Y_{CC}=2.7389$, $W_{NN}=2.45\times10^{-8}$ atm-s, and $Y_{NN}=14.3098$. Then the parallel resistance mixture rule given by Eqs. (3-5) can be written in the form

$$\tau_a'=(\tau_a')_{CC}/K_a,\qquad \tau_b'=(\tau_b')_{NN}/K_b,\qquad \tau_c'=(\tau_c')_{CC}/K_c$$

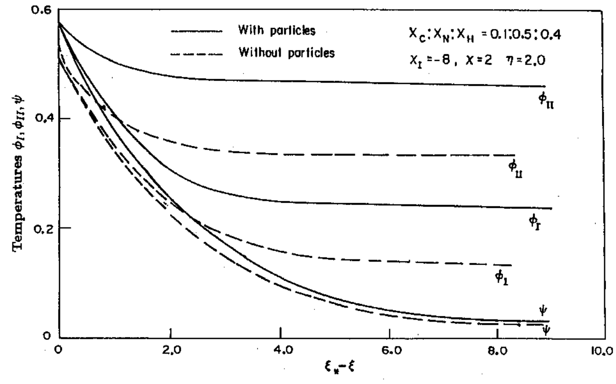

Fig. 3 Translational and vibrational temperature profiles along the nozzle axis.

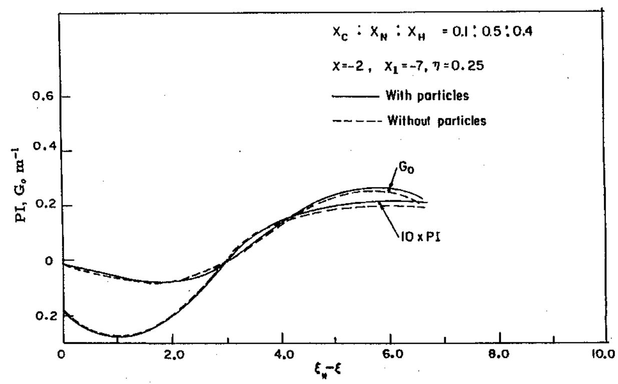

Fig. 4 Population inversion and small signal gain profiles along the nozzle axis.

where the various K are

$$K_a=X_C+X_N\left[\frac{(\tau_a')_{CC}}{(\tau_a')_{CN}}\right]+X_H\left[\frac{(\tau_a')_{CC}}{(\tau_a')_{CH}}\right]$$
$$K_b=X_N+X_C\left[\frac{(\tau_b')_{NN}}{(\tau_b')_{NC}}\right]+X_H\left[\frac{(\tau_b')_{NN}}{(\tau_b')_{NH}}\right]$$
$$K_c=X_C+X_N\left[\frac{(\tau_c')_{CC}}{(\tau_c')_{CN}}\right]+X_H\left[\frac{(\tau_c')_{CC}}{(\tau_c')_{CH}}\right]$$

Therefore, τ_{I}' and τ_{II}' can be written as

$$\tau_{\mathrm{I}}'=(\tau_c')_{CC}/K_{\mathrm{I}}$$
$$\tau_{\mathrm{II}}'=(\tau_b')_{NN}/K_{\mathrm{II}} \tag{27}$$

where, since $\tau_a'\gg\tau_c'$,

$$K_{\mathrm{I}}=K_c+K_a(\tau_c')_{CC}/(\tau_c')_{CC}\approx K_c$$
$$K_{\mathrm{II}}=[X_NK_b+X_CK_a(\tau_b')_{NN}/(\tau_a')_{CC}(X_C+X_N)^{-1}]$$

Equations (27) for the various τ_k' can be written in normalized form as

$$\tau_k'=\frac{W_{ll}}{p_0'\theta_N\psi K_k}\exp(\alpha+Y_{ll}\psi^{-1/3}),\qquad k=\mathrm{I},\ \mathrm{II} \tag{28}$$

where $l=C$ for $k=\mathrm{I}$ and $l=N$ for $k=\mathrm{II}$.

Using this equation and normalized temperature functions, the vibrational rate equations (16) can be written as

$$\frac{d\phi_k}{d\alpha}=\frac{K_k\psi}{N_s}Z^{-1/ij}u^{-1+1/ij}\exp[\lambda_k-\alpha(1-1/ij)-Y_{ll}\psi^{-1/3}]$$

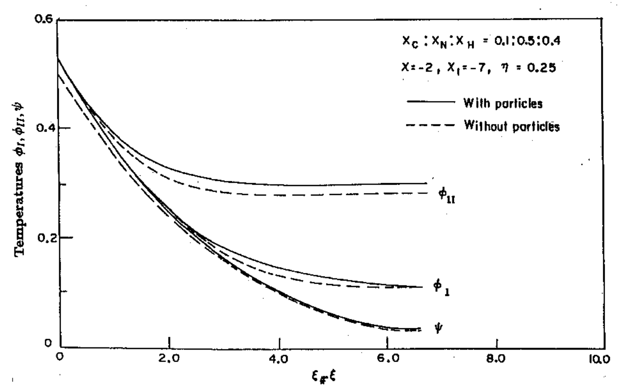

Fig. 2 Translational and vibrational temperature profiles along the nozzle axis.

$$\times\left(\frac{\bar{E}^{eq}-\bar{E}}{\bar{G}}\right)_k, \qquad k = \mathrm{I, II} \tag{29}$$

where

$$\lambda = \ell n\left[\frac{(\rho_* u_*)^{1/ij}}{ij\tau_v}\right]$$

$$\lambda_k = \ell n\left[\frac{p_0' L' \theta_N (\rho_* u_*)^{1/ij}}{iju_0' W_{ll}}\right], \qquad k = \mathrm{I, II}$$

$$\bar{G}_{\mathrm{I}} = \left[\frac{e^{\bar\theta_2/\phi_{\mathrm{I}}}(\bar\theta_1/\phi_{\mathrm{I}})^2}{(e^{\bar\theta_1/\phi_{\mathrm{I}}}-1)^2} + 2\,\frac{e^{\bar\theta_2/\phi_{\mathrm{I}}}(\bar\theta_2/\phi_{\mathrm{I}})^2}{(e^{\bar\theta_2/\phi_{\mathrm{I}}}-1)^2}\right]$$

$$\bar{G}_{\mathrm{II}} = \left[\frac{e^{\bar\theta_3/\phi_{\mathrm{II}}}(\bar\theta_3/\phi_{\mathrm{II}})^2}{(e^{\bar\theta_3/\phi_{\mathrm{II}}}-1)^2} + \frac{X_N}{X_C}\frac{1}{\phi_{\mathrm{II}}^2}\frac{e^{1/\phi_{\mathrm{II}}}}{(e^{1/\phi_{\mathrm{II}}}-1)^2}\right]$$

$$\bar{E}_{\mathrm{I}} = \frac{\bar\theta_1}{e^{\bar\theta_1/T_{\mathrm{I}}}-1} + \frac{2\bar\theta_2}{e^{\bar\theta_2/\phi_{\mathrm{I}}}-1}$$

$$\bar{E}_{\mathrm{II}} = \frac{\bar\theta_3}{e^{\bar\theta_3/\phi_{\mathrm{II}}}-1} + \frac{X_N/X_C}{e^{1/\phi_{\mathrm{II}}}-1}$$

Equations (23–26) and (29) form the final set of equations to be solved for six variables: u, u_p, ψ, ψ_p, ϕ_{I}, and ϕ_{II}.

In almost all practical cases of nozzle flows, the flow is assumed to be in equilibrium (vibrationally) up to the throat. The aim here is to study the effect of the particles on the optimum gain. Since the PI starts building up only in the downstream of the throat region, the nonequilibrium between the gas and particles is also considered in the downstream region of the throat.

As observed earlier, to obtain the nonequilibrium solution from Eqs. (23–26) and (29), it is necessary to specify the initial values for the dependent variables at a given value of η. Since the flow in a GDL starts from a reservoir wherein the hot gas mixture is in thermodynamic equilibrium, the initial conditions would correspond to this state. However, by employing an equilibrium solution as an initial condition, it is proved[19] that the solution depends on another parameter, the reservoir entropy S_0; i.e., S_0 appears as a parameter of the problem in addition to the parameters (ij), λ_k, and λ. The current investigation is to reduce the parameter dependence of the nonequilibrium solutions to a minimum and hence to obtain better inference of the performance of a GDL. Therefore, another transformation is introduced here to reduce the number of parameters. A new independent variable ξ is introduced for this purpose and is defined as $\xi = (S_0 - \alpha)$. In terms of the independent variable ξ, Eqs. (23–26) and (29) are

$$u\frac{du}{d\xi} + \eta u\frac{du_p}{d\xi} + \theta_N\frac{d\psi}{d\xi} + \theta_N\psi = 0 \tag{30}$$

$$u\frac{du}{d\xi} + (1+\epsilon)\theta_N\frac{d\psi}{d\xi} + X_C\sum_{k=\mathrm{I}}^{\mathrm{II}}\overline{G_k}\theta_N\frac{d\phi_k}{d\xi} + \eta\left(\frac{du_p}{d\xi} + \frac{\gamma}{\gamma-1}\,\delta\theta_N\frac{d\psi_p}{d\xi}\right) = 0 \tag{31}$$

$$\frac{du_p}{d\xi} = \frac{e^{\chi-\xi/ij}}{N_s}\,\frac{u-u_p}{u_p}\,f(Re)(uZ)^{-1/ij} \tag{32}$$

$$\frac{d\psi_p}{d\xi} = -\frac{e^{\chi-\xi/ij}}{N_s}\,\frac{1}{\delta}\,\frac{\psi-\psi_p}{u_p}\,\frac{Nu(Re)}{2}\,(uZ)^{-1/ij} \tag{33}$$

$$\frac{d\phi_k}{d\xi} = -\frac{K_k\psi}{N_s}Z^{-1/ij}u^{-(1+1/ij)}\exp[\chi_k + \xi(1-1/ij) - Y_{ll}\psi^{-1/3}]\left(\frac{\bar{E}^{eq}-\bar{E}}{\bar{G}}\right)_k, \qquad k = \mathrm{I, II} \tag{34}$$

where

$$\chi = \lambda + S_0/ij$$

$$\chi_k = \lambda_k - S_0(1 - 1/ij)$$

$$N_s = \left(\frac{M_v^2}{M_v^2-1}\right)(1 - A_v^{-1/i})^{(j-1)/j}$$

An examination of this set of governing equations reveals that, for a given value of (ij) and a given laser gas mixture, the solution will depend on three parameters χ_{I}, χ_{II}, and χ, the first two characterizing the vibrational relaxation of different modes in the gas and the third characterizing the particle properties. However, it can be shown[22] that $\chi_{\mathrm{II}} = \chi_{\mathrm{I}} + \ell n(W_{CC}/W_{NN})$ where W_{CC} and W_{NN} are constants. Therefore, the two-phase flow GDL problem has finally been reduced to depend on two parameters, χ_{I} and χ (characterizing vibrational relaxation phenomena and particle relaxation, respectively). Simple expressions for the functions $f(Re)$ and $Nu(Re)$ (appearing in the drag and heat transfer rate equations) have been used here. The nonsimilar function N_s has been correlated[19] accordingly and has been used for the current investigation.

These equations have been solved using the Runge-Kutta-Gill method. Knowing the nonequilibrium solution along the nozzle, the gain G_0 for the $P(20)$:(001)→(100) transition has been estimated using the following expression:

$$G_0 = 0.0977\,\frac{e^{\bar\theta_3/\phi_{\mathrm{II}}} - e^{\bar\theta_1/\phi_{\mathrm{II}}}}{Q_{\mathrm{vib}}P(X)\psi^{3/2}}\,e^{-0.0703/\psi} \tag{35}$$

This is a modified form of Eq. (10), and $\lambda_\nu = 10.6\ \mu$m is the wavelength of the laser beam, $P(X) = 1 + 0.7589(X_N/X_C) + 0.6972(X_H/X_C)$.

Results and Discussions

For a given gas mixture, the equations are solved numerically for the unknowns using the fourth-order Runge-Kutta-Gill method. The results are presented in the form of graphs. As mentioned earlier, the parameters χ and χ_{I} contain all of the important parameters of the problem and hence control the characteristics of the solutions. The parameter χ, which controls the particle characteristics (through τ_v) introduced in Eqs. (32) and (33), is given by

$$\chi = \ell n\left[\frac{(\rho_* u_*)^{1/ij}}{(ij)\tau_v}\right] + S_0/ij$$

It can be inferred that a small value of χ corresponds to particles of large size and vice versa. It is known from an earlier investigation[19] that large particles (which correspond to small χ) present in the flow will freeze at the initial conditions and hence have no effect on the gas, whereas the small particles (which correspond to large χ) are almost in equilibrium with the gas during the expansion process and show pronounced effects on the gas. Hence, the variation in the value of χ scans the complete nonequilibrium flow regime (between gas and particles) from equilibrium to frozen conditions.

Figures 2 and 3 show the profiles of the translational temperatures ψ and vibrational temperatures ϕ_{I} and ϕ_{II} of (lower and upper) modes I and II, respectively, along the nozzle starting from the nozzle throat for $\eta = 0.25$ and 2. It is clear that during the nozzle expansion process the vibrational energies relax at a slower rate than the translational energy. The vibrational temperatures of the two modes of the lasing gas mixture try to equilibrate with the translational temperature and in the process freeze; the vibrational temperature corresponding to mode II of the lasing gas mixture freezes earlier than the vibrational temperature of mode I. The level of freezing is affected by the presence of the particles. It is also clear from these figures that the vibrational temperatures corresponding to the two modes freeze at a higher level than that

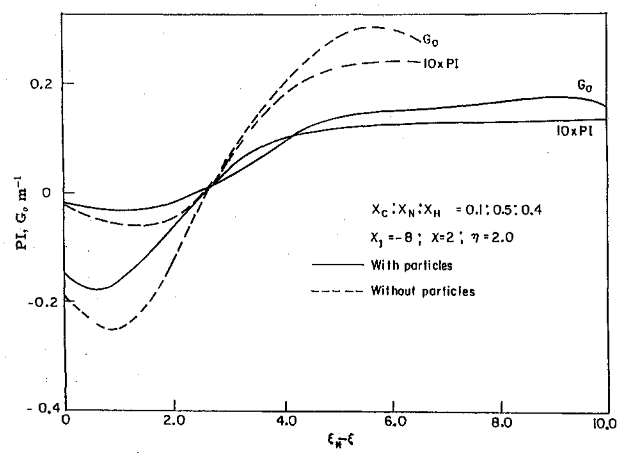

Fig. 5 Population inversion and small signal gain profiles along the nozzle axis.

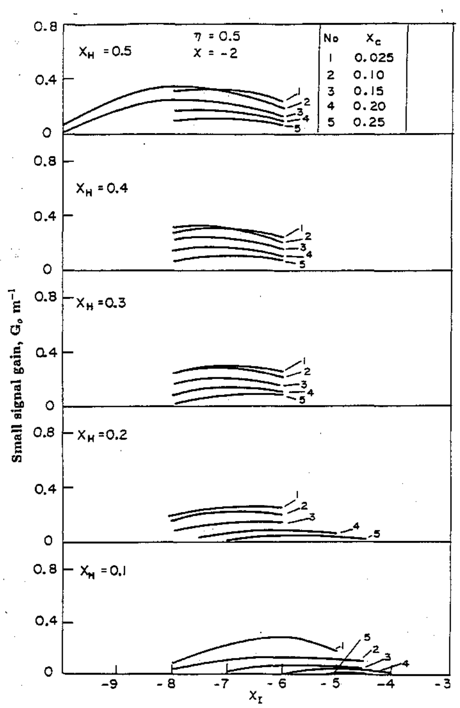

Fig. 6 Maximum values of the small signal gain as a function of the parameter χ_I.

for a pure gas case as the loading ratio is increased. Although the vibrational temperatures freeze at higher levels in the presence of particles, the difference between the vibrational temperatures decreases as η increases. This can possibly be explained as follows: At high loadings, when energy is transferred to gas from particles due to heat transfer, the vibrational energy levels get populated more and more, with lower levels getting populated more than the higher levels. Physically, this means that the population inversion is decreased as loading ratio is increased. Hence, the difference between the vibrational temperatures is small, whereas the vibrational temperatures themselves are large. This effect of particles on the vibrational temperature of the two modes is expected to show up adversely in the PI and hence on the performance of the GDL. The PI and the gain G_0 profiles are presented in Figs. 4 and 5 for $\eta = 0.25$ and 2. It can be seen from these figures that the gain attains a peak value $(G_0)_{max}$ after which it starts decreasing, whereas PI reaches a constant value far downstream of the nozzle throat. The difference between the maximum gains that can be obtained when no particles are present in the gas and when large particles are present at low loadings is insignificant as shown in Fig. 4. At a large χ, Fig. 5 shows the comparison of the results without and with particles at $\chi = 2$. The maximum value of G_0 obtained in the case of the gas-particle mixture has been significantly reduced compared with the pure gas case. This shows that either increase in loading ratio or increase of χ (corresponding to the decrease of particle size) or both could be the reason for the effective decrease in gain. In all of these four figures the broken lines represent the flow variables when no particles are present. Hence, the effect of the presence of the particles could easily be seen from these four figures.

The peak gain values as a function of χ_I for a given χ and η are computed and plotted in Fig. 6 for a set of gas compositions. These curves show that $(G_0)_{max}$ reaches a second peak that is called an optimum (maximum of maxima) value designated as $(G_0)_{opt}$ at a certain χ_I that is called $(\chi_I)_{opt}$ for a given composition and χ.

Figure 7 shows the optimum gain plotted as a function of the loading ratio η at different χ. As η is increased, the drastic decrease in optimum gain achieved could be noticed, and at much larger η (4 or 5), the optimum gain never reaches a positive value whatever may be the value of χ (and hence the particle size). That is, even if the size of particles present in the flow is large enough to cause the particles to remain in a near-frozen condition, positive gain in a two-phase flow GDL system at high particle loading conditions cannot be expected. In other words, the effect of the contaminants is so adverse that the whole effort of getting a high-power laser beam will be fruitless. This observation has been made earlier by Veselov and Markachev.[18] However, the problem has been studied[18] with a different geometrical setup of nozzles, as mentioned earlier, in which the highly excited N_2 is mixed with the cold CO_2–He mixture downstream of the nozzle exit. Moreover, no optimization of the performance is carried out in that study.[18] Figure 7 also shows that the gain varies only insignificantly for a small range of η (up to $\eta = 0.5$) for different gas compositions as was observed earlier.[23] This is true for almost all values of χ presented except for very large χ. For large values of χ the effect is only little. Hence, it may be concluded that, whatever may be the gas composition, for small values of χ,

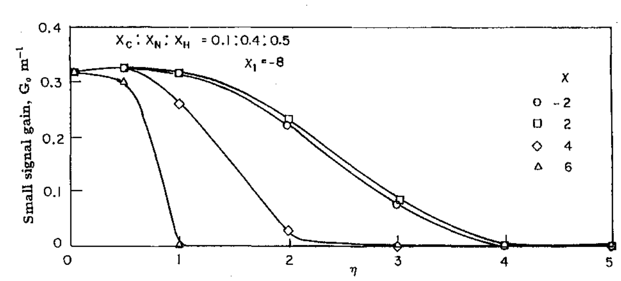

Fig. 7 Optimum small signal gain as a function of the loading ratio.

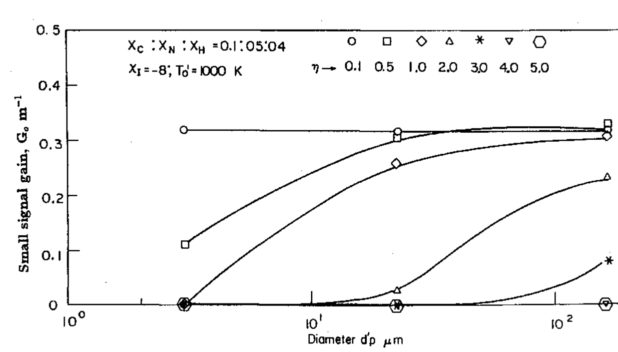

Fig. 8 Optimum small signal gain as a function of the particle diameter.

the particles present in the flow will not affect the nozzle flow expansion process and thereby do not affect the PI and gain values.

Figure 8 presents the gain as a function of particle diameter at various loading ratios. As the particle size is decreased, the decrease in gain becomes significant. For smaller η, this effect is not much since the amount of particles present at any given cross section of the nozzle is very small. As η is increased, the gain values constantly decrease and eventually tend to zero at higher η values. Therefore, the naturally present contaminants in the gas mixture are expected to affect the lasing performance of a GDL adversely.

Conclusions

An optimum gain calculation study of a two-phase flow gasdynamic laser has been carried out successfully. The gain calculation has been optimized using the fact that the particles of large size do not affect the flow expansion process significantly. The effect of variation of particle characteristics on the optimum gain has then been studied. For a given small loading ratio, the effect of the presence of the particles on the gain is less when the particle size is large, and as the size is decreased, the effect becomes more and more pronounced. As the loading ratio is increased, the gain decreases for all values of the correlating parameter χ, and at larger loading ratios, the gain reaches zero, implying that no lasing action is taking place. Therefore, it may be concluded that the naturally present very small particles or large loading of particles will have an adverse effect on the optimum performance of a GDL.

Appendix

The existing theoretical analyses of gas-particle nozzle flows use the speed of sound pertaining either to equilibrium or frozen limits between the two phases. Since the nozzle flow has to be treated as a two-phase flow expansion process, as mentioned earlier, the thermodynamic properties of the corresponding mixture have to be used for any computations. Therefore, the use of definitions of the speed of sound as available for equilibrium or frozen flow of a gas-particle mixture in the current problem of nonequilibrium two-phase flow is not suitable. For the present work, new definitions for effective speed of sound and effective Mach number, which are called "virtual speed of sound" and "virtual Mach number" have been introduced[19] for the case of nonequilibrium gas-particle nozzle flows. Using these definitions, simple area ratio Mach number relations that are very similar to those of the pure gas case have been deduced[19] from continuity and momentum equations, and one of them is given by

$$\frac{\mathrm{d}(uZ)}{uZ} = \frac{\mathrm{d}(A_v)/A_v}{(M_v^2 - 1)}$$

where $M_v = u/a_v$, $a_v = [(\mathrm{d}p/\mathrm{d}\sigma)/Z]^{1/2}$, and $A_v = A/Z$. From this new expression of speed of sound for nonequilibrium gas-particle nozzle flow, the existing expressions for speed of sound are deduced under the limiting conditions of equilibrium or frozen flow.

Use of this new set of expressions is crucial in obtaining the expression for $(\mathrm{d}\alpha/\mathrm{d}x)$ given earlier and subsequent use of this expression in obtaining similar solutions. Derivation of all of these expressions is available in Ref. 19.

Acknowledgment

The first author acknowledges gratefully the Council of Scientific and Industrial Research of the government of India for the financial support made available to him through the award of a senior research fellowship in engineering.

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